Solutions to JEE MAIN – 4 | JEE 2024

PHYSICS

SECTION-1

1.(A) $\tan\phi' = \frac{1}{\cos\theta} \times \tan\phi$

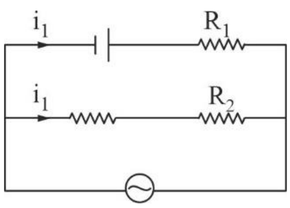
$\tan 60° = \frac{1}{\cos 45°} \times \tan\phi$

Actual dip $\phi = \tan^{-1}\left(\sqrt{\frac{3}{2}}\right)$

2.(A) The potential difference across the inductor is $e = E - iR$

Hence the plot of e versus i is a straight line with negative slope.

3.(C) $i_{1rms} = \frac{E_{rms}}{\sqrt{X_c^2 + R_1^2}} = \frac{130}{13} = 10A$



$i_{2rms} = \frac{E_{rms}}{\sqrt{X_L^2 + R_2^2}} = 13A$

Power dissipated

$i_{1rms}^2 R_1 + i_{2rms}^2 R_2 = 10^2 \times 5 + 13^2 \times 6 = 1514W$ = power delivered by battery

4.(A) Induced emf in the rod $\varepsilon = Blv$

Current in the circuit $I = \frac{\varepsilon}{R} e^{-t/RC}$

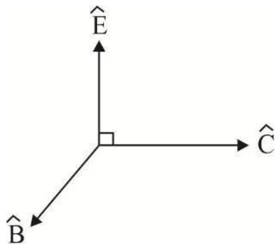
Since the net force on the rod should be zero, the external force will be equal in magnitude but opposite to the magnetic force.

$\Rightarrow \quad F = BIl = \frac{B^2 l^2 v}{R} e^{-t/RC}$

5.(B) $\hat{c} = E \times B$

$\Rightarrow \quad \hat{c} = \hat{j} \times \hat{i}$

$\therefore \quad \hat{c} = -k$



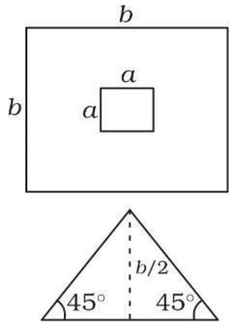
6.(D) Conceptual

7.(B) $T = 2\pi\sqrt{\dfrac{I}{MB_H}}$; $\dfrac{M_1}{M_2} = \dfrac{I_1}{I_2} \times \left(\dfrac{T_2}{T_1}\right)^2 = \dfrac{8}{3}$

8.(B) Magnitude of electric field, $E_0 = B_0 c = (1.2 \times 10^{-7})(3 \times 10^8) = 36\ V/m$

If the direction of propagation is along the unit vector $\hat{S}$, the direction of electric field is along $\hat{B} \times \hat{S}$. Therefore, in the given situation, the electric field is along –Y direction.

9.(D) $B = 4 \times \dfrac{\mu_0 i}{4\pi \frac{b}{2}} \dfrac{2}{\sqrt{2}} = \dfrac{2\sqrt{2}\mu_0 i}{\pi b}$



$BA = Mi$

$\dfrac{2\sqrt{2}\mu_0 i}{\pi b} \times a^2 = Mi$

$M = \dfrac{2\sqrt{2}\mu_0}{\pi b} a^2 = \dfrac{\mu_0}{4\pi} 8\sqrt{2} \dfrac{a^2}{b}$

Flux = $MI = \dfrac{\mu_0}{4\pi} 8\sqrt{2} \dfrac{a^2}{b} I$

10.(D) Current is going in the opposite sense in the other coil

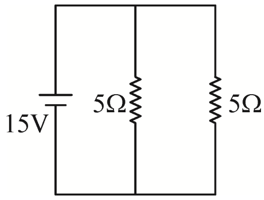
$\therefore \quad L_{eq} = L_1 + L_2 - 2M$

11.(C) $i = i_1 \sin\omega t + i_2$

$\left\langle i^2 \right\rangle = \dfrac{i_1^2}{2} + i_2^2 + 0$

$RMS = \left(\dfrac{i_1^2}{2} + i_2^2\right)^{1/2}$

12.(A) $I = \dfrac{15}{2.5} = 6A$



13.(B) Gamma rays used in Cancer Treatment
X-rays used as Diagnostic tool in medicine
Microwave used for Communication, Radar
Radio wave used for TV Communication system

14.(A) $\dfrac{V_P}{V_S} = \dfrac{N_P}{N_S}$; $\dfrac{8000}{80} = \dfrac{N_P}{N_S}$

15.(A) $B = \dfrac{\mu_0 M}{4\pi R^3}$

$M = \dfrac{4\pi}{\mu_0} BR^3 = 1.5 \times 10^{23} A - m^2$

16.(B) $\int \vec{E}.d\vec{\ell} = \frac{d\phi_B}{dt}$

$E\left(2\pi \times \frac{1}{2}\right) = \pi\left(\frac{1}{2}\right)^2 \times \frac{dB}{dt}$

$E(1) = \frac{1}{4} \times 4t$

$E(t) = t$

$E(2) = 2N/C$

17.(D) $\chi = \frac{C}{T}$; $\quad 2.5 \times 10^{-4} = \frac{C}{350}$ ... (i)

$\chi = \frac{C}{250}$ ... (ii)

$\chi = 3.5 \times 10^{-4}$

18.(B) $\lambda_{\text{gamma - ray}}$ is shortest

$\lambda_{\text{AM Radio}}$ is largest

19.(D) Soft iron has high permeability.

20.(B) Let Q denote maximum charge on capacitor.
Let q denote charge when energy is equally shared

$\therefore \ \frac{2}{3}\left(\frac{1}{2}\frac{Q^2}{C}\right) = \frac{1}{2}\frac{q^2}{C} \Rightarrow \quad \therefore \ q = \frac{\sqrt{2}}{\sqrt{3}}Q$

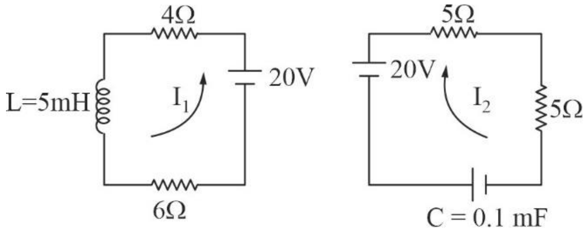
SECTION-2

1.(100) Under resonance $i = \frac{V_0}{R} = 5A$ Voltmeter reading $V_0 = 500V$

2.(175) $\frac{f_{\max}}{\Delta f_{half \max power}} = \text{Quality factor}$

$\frac{X_C}{R} = \text{Quality factor}$

3.(5) $I_1 = \frac{20}{10}\left(1 - e^{-\frac{10t}{5\times10^{-3}}}\right) = \frac{3}{2} = 1.5A$



$I_2 = \frac{20}{10}e^{-\frac{t}{10\times0.1\times10^{-3}}} = 1.0A$

From superposition $I = I_1 + I_2 = 2.5A$

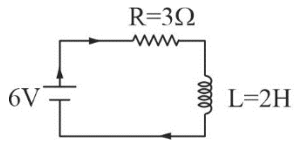
4.(10) $\phi_E = \vec{E}.\vec{A} = \left(\frac{V}{d}\right)A = \frac{(10\sin\omega t)}{d}A$

Also, $C = \frac{\varepsilon_0 A}{d}$; $I_d = \varepsilon_0\left(\frac{d}{dt}\phi_E\right) = \left(\varepsilon_0\frac{A}{d}\right)10\omega\cos\omega t$

Amplitude of $I_d = \frac{8.85\times10^{-12}\times10/\pi}{8.85\times10^{-3}}\times10\times2\pi\times50 = 10\times10^{-6} = 10\mu A$

5.(4) 1st circuit

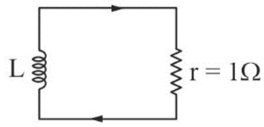
At steady state



$i = i_0 = \frac{6}{R} = \frac{6}{3} = 2A$

2nd circuit

Initial current $= i_0$



Energy loss = 1/2 L i^2 = 1/2 × 2 × 4 = 4 J

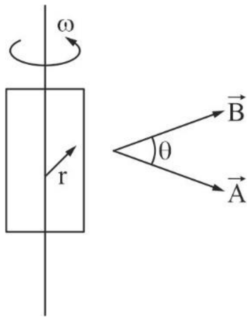
6.(192) $\phi = \vec{B}\cdot\vec{A} = BA\cos\omega t$

$$-\frac{Nd\phi}{dt} = NBA\omega\sin\omega t = E$$

$\Rightarrow \quad E_{\max} = NBA\omega = a^2\times B\times\omega\times N$

$$= \frac{50}{100}\times\frac{64}{100}\times3\times10^{-2}\times20$$

$$= 19.2\times10^{-2}\ volts$$



7.(1) $e = \frac{-d\phi}{dt}$

$e = -(16t - 10)$

At $t = 0.5$ $\quad e = 2V$

$I = \frac{e}{R}$; $I = \frac{2}{2}A$

8.(2) Coercivity of ferromagnet $H = 200\ A/m$ $nI = 200$

$I = \frac{200}{100} = 2A$

9.(4) $n = \frac{c}{V} = \frac{3\times10^8}{1.5\times10^8} = 2$

$n = \sqrt{\mu_r \in_r}$; $\in_r = n^2 = \boxed{4}$

10.(3) $H_{(DC)} = I_{DC}^2\times R = (4)^2\times3 = 16\times3J$

$H_{(AC)} = (I_{rms})^2\times R' = (4/\sqrt{2})^2\times2 = 16\,J$

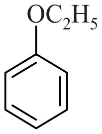
$\therefore$ ratio of heat produces is 3 : 1

CHEMISTRY

SECTION-1

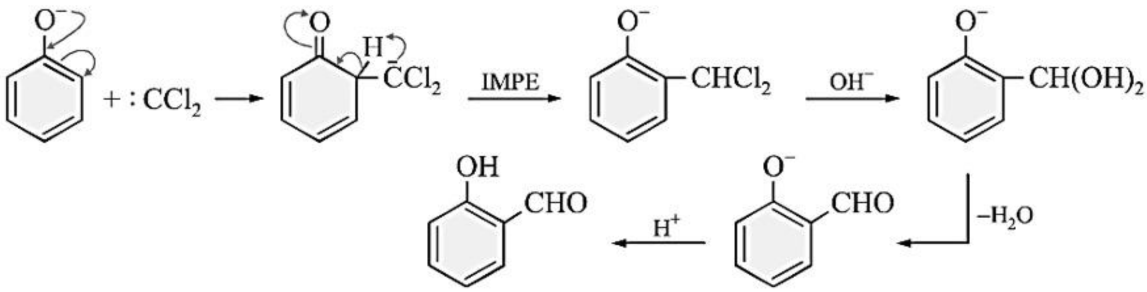
1.(C) Methanol is used as a solvent in paints
Methanol is known as wood spirit
Invertase convert sugar into glucose & fructose
Hence, statements (A) and (B) are incorrect.

2.(B) Phenetole is $C_6H_5OCH_2CH_3$

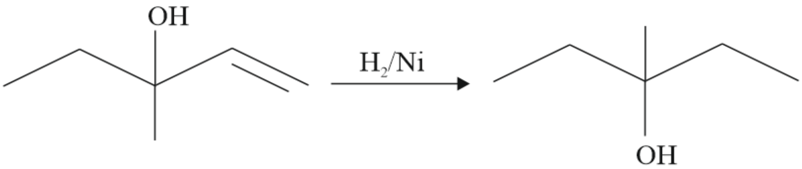


: Ethoxy benzene

3.(C) $OH^- + H-CCl_3 \xrightarrow[-H_2O]{} {}^-CCl_3 \xrightarrow{\Delta} :CCl_2 + Cl^-$

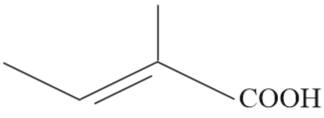


4.(B or C)



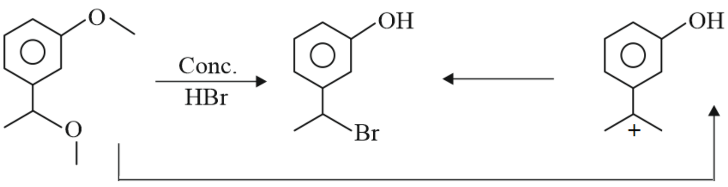
5.(C) DIBAL – H convert R – CN to RCHO.

6.(D) $CH_3-CH=\underset{\underset{CH_3}{|}}{C}-\overset{\overset{O}{||}}{C}-CH_3 \xrightarrow[H_2O]{NaOCl}$



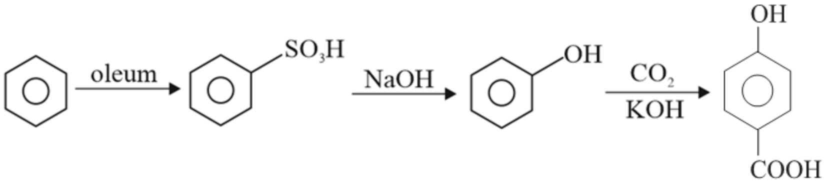
(Haloform Reaction)

7.(C)



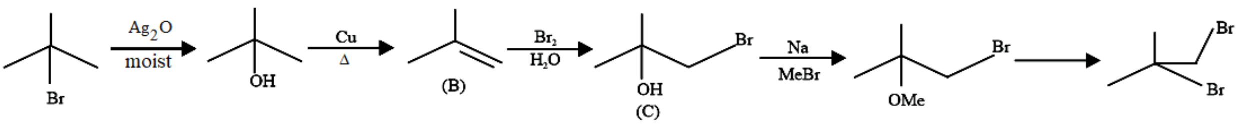
8.(D) Phenol is more acidic than alcohol, so it will form phenoxide ion.

9.(A)



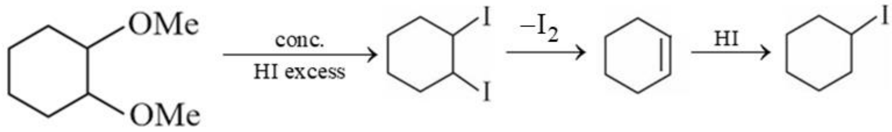
10.(D) Bond angle of ether is $\approx 110°$.

11.(C)

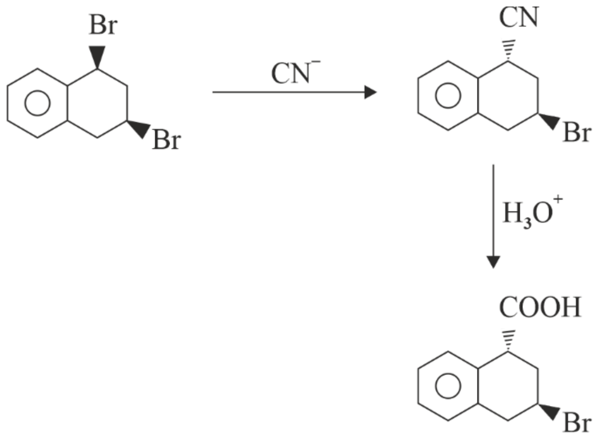


12.(B) $Ph - CN \xrightarrow[H_2O]{DIBAL-H} PhCHO$

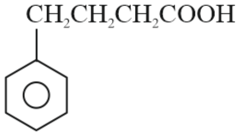
13.(D)

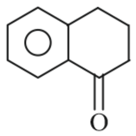


14.(B)

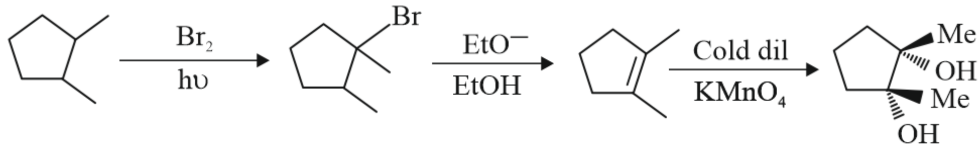


15.(B) Rate of S_N1 depends on stability of carbocation while rate of S_N2 depends on approachability (less steric hinderance)

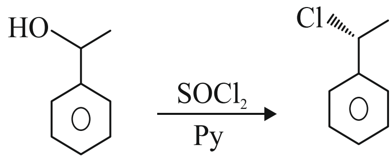
16.(B) P = 

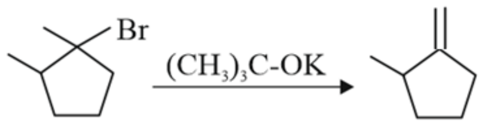
Q = 

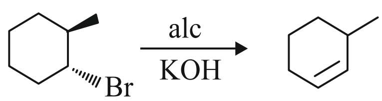
17.(D)



18.(A)

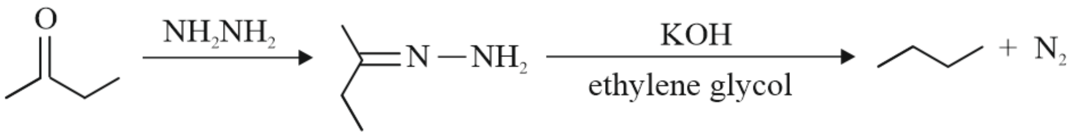
(I) 

(II) 

(III) 

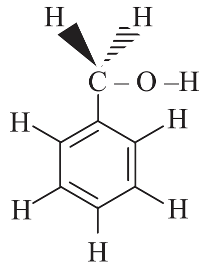
(IV) Not Possible

19.(B) I-Q ; II – S ; III – R ; IV - P

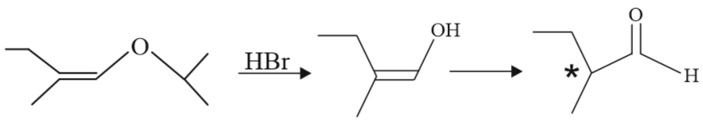
20.(B) 

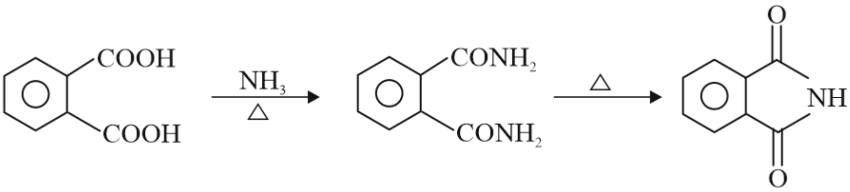
SECTION-2

1.(14) $Ph-Et \xrightarrow[\Delta]{\text{alk. } KMnO_4} Ph-COOH \xrightarrow[H_3O^+]{B_2H_6} Ph-CH_2OH$

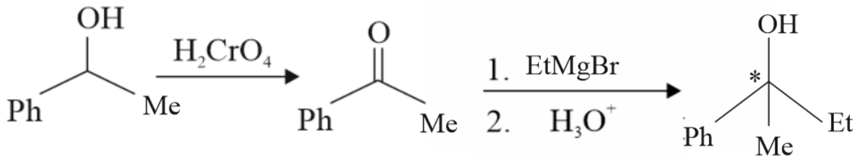


2.(4) NaOH and CaO are used in ratio of 3 : 1.

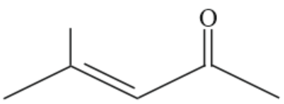
3.(2) 

4.(7) 

5.(2)



6.(3) Salicylaldehyde, Acrolein, Cinnamaldehyde

7.(3) Mesityl oxide is 

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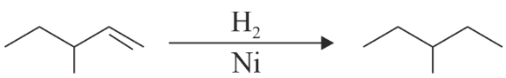
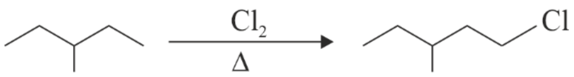
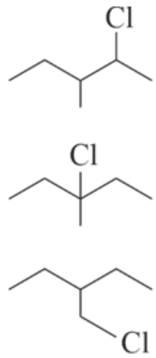
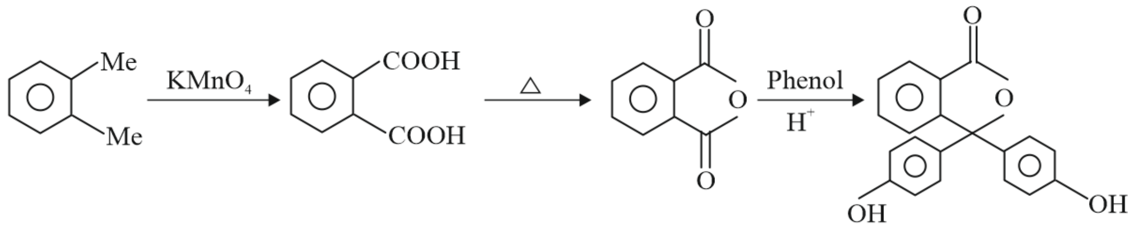
8.(4) 





9.(14)



Phenolphthalein

10.(3) Hydrazine, Grignard reagent, 2, 4 DNP

MATHEMATICS

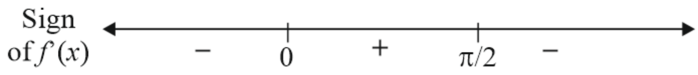
SECTION-1

1.(D) $f'(x) = 12\sin^3 x\cos x + 6\cos^5 x.\sin x$

$= 6\sin x\cos x(2\sin^2 x + \cos^4 x)$

$= 3\sin 2x(1+\sin^4 x)$

$f'(x) = 0 \Rightarrow \sin 2x = 0 \Rightarrow x = 0, \frac{\pi}{2}$



Minimum value of $f(x) = f(0) = -1$

Maximum value of $f(x) = f\left(\frac{\pi}{2}\right) = 3$

2.(B) $f''(x) = 6x - 6$

$\Rightarrow f'(x) = \int (6x-6)\,dx = 3x^2 - 6x + C$

$\Rightarrow f'(x)\,|\,(2,1) = C = 3 \Rightarrow C = 3$

$\Rightarrow f'(x) = 3x^2 - 6x + 3$

$\Rightarrow f(x) = x^3 - 3x^2 + 3x + A \xrightarrow{(2,1)} 1 = 8 - 12 + 6 + A \Rightarrow A = -1$

$\therefore f(x) = x^3 - 3x^2 + 3x - 1 = (x-1)^3$

3.(A) Length of Subnormal $= y\frac{dy}{dx}$

$= \left(3^{1-k}\cdot x^k\right)\left(3^{1-k}\cdot kx^{k-1}\right)$

$= 9^{1-k}\cdot K\cdot x^{2k-1} = \text{constant} \Rightarrow 2k-1 = 0$

4.(D) $I = \int\limits_{-(\pi/2)}^{\pi/2} \frac{\cos^2 x}{1+5^x}dx = \int\limits_{-(\pi/2)}^{\pi/2} \frac{\cos^2 x}{1+5^{-x}}dx = \int\limits_{-(\pi/2)}^{\pi/2} \frac{5^x\cos^2 x}{5^x+1}dx$

$2I = \int\limits_{-(\pi/2)}^{\pi/2} \cos^2 x\,dx = \int\limits_{-(\pi/2)}^{\pi/2} \frac{1+\cos 2x}{2} = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$

5.(A) $\int \frac{8x^{43}+13x^{38}}{(x^{13}+x^5+1)^4}dx = \int \frac{\frac{8}{x^9}+\frac{13}{x^{14}}}{\left(1+\frac{1}{x^8}+\frac{1}{x^{13}}\right)^4}dx$

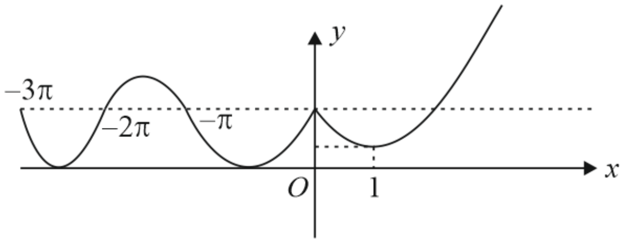
$= \frac{1}{3\left(1+\frac{1}{x^8}+\frac{1}{x^{13}}\right)^3} + C = \frac{x^{39}}{3(x^{13}+x^5+1)^3} + C$

6.(A) $\frac{1}{2}\int\limits_0^1 x\cdot(2x\cdot e^{-x^2})dx = \frac{1}{2}\left[\left(-xe^{-x^2}\right)_0^1 + \int\limits_0^1 e^{-x^2}dx\right]$

$= \frac{1}{2}\left[-\frac{1}{e} + a\right]$

7.(D) $f(0^-) \geq f(0) \Rightarrow a \geq 3$

8.(A)



$f(x)$ has local maximum at $x = 0$

9.(D) $\int \frac{dx}{(x-1)^{3/4}(x+2)^{5/4}} = \int \frac{dx}{\left(\frac{x-1}{x+2}\right)^{3/4}(x+2)^2}$

Let $\frac{x-1}{x+2} = t \Rightarrow dt = \frac{3dx}{(x+2)^2} = \frac{1}{3}\int \frac{dt}{t^{\frac{3}{4}}} = \frac{1}{3}\frac{t^{1/4}}{\frac{1}{4}} + C = \frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{1}{4}} + C$

10.(B) $\lim_{x\to\infty} \frac{1}{n}\sum_{r=1}^{n} \frac{1}{\sqrt{1+\frac{r}{n}}} = \int_0^1 \frac{dx}{\sqrt{x+1}} = 2(\sqrt{2}-1)$

11.(B) $f(x) = \int_0^2 e^{|x-t|}dt$

$\Rightarrow f(x) = \int_0^x e^{x-t}dt + \int_x^2 e^{t-x}dt = \left(-e^{x-t}\right)_0^x + \left(e^{t-x}\right)_x^2$

$\Rightarrow f(x) = -e^{x-x} + e^x + e^{2-x} - e^{x-x} = -2 + e^x + \frac{e^2}{e^x}$

Min value of $f(x) = -2 + \min$ value of $e^x + e^{2-x}$ $= -2 + 2e$

12.(C) We prepare the table as given below:

Age (in years) Number of student 1;, xi

Age (in years)((x_i)	Number of students f_i	$f_i x_i$
12	6	72
13	8	104
14	5	70
15	7	105
16	9	144
17	5	85
	$\Sigma f_i = 40$	$\Sigma f_i x_i = 580$

$\therefore$ Mean age $= \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{580}{40} = 14.5\, years$

13.(D) For $0 < x < 1$

$1+x^9 < 1+x^8 < 1+x^4 < 1+x^3$

$\frac{1+x^8}{1+x^4} < 1,\ \frac{1+x^9}{1+x^3} < 1,\ \frac{1+x^8}{1+x^4} > \frac{1+x^9}{1+x^3}$

14.(A) Let general point on the curve $y = \sqrt{x}$ be $(t^2, t), t > 0$

Distance $= \sqrt{(t^2-2)^2 + (t-1)^2}$

$D = (t^2-2)^2 + (t-1)^2 = t^4 - 3t^2 - 2t + 5$

$\frac{dD}{dt} = 4t^3 - 6t - 2 = 0 \Rightarrow t = \frac{1+\sqrt{3}}{2}$

$\left.\frac{d^2D}{dt^2}\right|_{t=\frac{1+\sqrt{3}}{2}} = 12t^2 - 6\Big|_{t=\frac{1+\sqrt{3}}{2}} > 0$

$\Rightarrow$ D is minimum at $t = \frac{1+\sqrt{3}}{2} \Rightarrow$ x-coordinate $= \frac{2+\sqrt{3}}{2}$

15.(D) $\frac{\sin\theta}{\theta}$ is decreasing function on $\left(0, \frac{\pi}{2}\right)$

$\frac{\theta}{\tan\theta}$ is decreasing function on $\left(0, \frac{\pi}{2}\right) \Rightarrow \frac{\sin\theta}{\theta} + \frac{\theta}{\tan\theta}$ is decreasing function on $\left(0, \frac{\pi}{2}\right)$

16.(A) $2I = \int_{-20\pi}^{20\pi} |\sin x|\left([\sin x] + [-\sin x]\right)dx = -\int_{-20\pi}^{20\pi} |\sin x|\,dx - 20\pi$

$I = -20\int_0^{\pi} \sin x = -40$

17.(C) Let the variable be *x*. Here *n* = 100. Incorrect *A* = 40, incorrect $\sigma = 5.1$

From formula $A = \frac{\Sigma x}{n} \Rightarrow \Sigma x = nA$

$\therefore$ Incorrect $\Sigma x = 100 \times 40 = 4000$

$\therefore$ Correct $\Sigma x =$ incorrect $\Sigma x - 50 + 40 = 4000 - 50 + 40 = 3990$

Again, from $\sigma^2 + A^2 = \frac{\Sigma x^2}{n}$,

We get $\Sigma x^2 = n(\sigma^2 + A^2)$

$\therefore$ Incorrect $\Sigma x^2 = 100\{(5.1)^2 + (40)^2\} = 100(26.01 + 1600) = 162601$

$\therefore$ Correct $\Sigma x^2 =$ incorrect $\Sigma x^2 - 50^2 + 40^2 = 162601 - 2500 + 1600 = 161701$

$\therefore$ Correct mean $A = \frac{\Sigma x}{n} = \frac{3990}{100} = 39.9$

Correct $(\sigma^2 + A^2) = \frac{\Sigma x^2}{n} = \frac{161701}{100} = 161701$

$\therefore$ Correct $\sigma^2 = 1617.01 - (\text{correct } A^2)$

$= 1617.01 - (39.9)^2 = 1617.01 - 1592.01 = 25$

18.(B) Mean $= \bar{x} = \dfrac{3+4+5+6+7}{5} = 5$

x	$\lvert x-\bar{x} \rvert$
3	2
4	1
5	0
6	1
7	2

$\therefore\ \Sigma \lvert x-\bar{x} \rvert = 6 \quad \Rightarrow$ Mean deviation $= \dfrac{6}{5} = 1.2$

19.(B) $\displaystyle \lim_{n\to\infty} \frac{1}{\sqrt{n}} \sum_{r=1}^{n} \frac{1}{\sqrt{r}} = \lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} \frac{1}{\sqrt{\frac{r}{n}}} = \int_0^1 \frac{1}{\sqrt{x}}\, dx = \left(2\sqrt{x}\right)_0^1 = 2$

20.(C) $\int_0^1 (1-x^7)^{1/4} dx - \int_0^1 (1-x^4)^{1/7} dx$

$y = f(x) = (1-x^7)^{1/4} \quad \Rightarrow \quad y^4 = (1-x^7) \quad \Rightarrow \quad x = (1-y^4)^{1/7}$

Hence functions are inverse of each other.

$I = \int_0^1 f(x)\, dx + \int_{f(0)}^{f(1)} f^{-1}(x) dx = 1 f(1) - 0 f(0) = f(1) = 0$

SECTION-2

1.(5) $g(x) = \dfrac{(|x|-1)(|x|-2)}{(|x|-3)(|x|-4)}$

$g(x)$ is an even function so there is an extrema at x = 0.

Also number of extrema for $x > 0$ will be equal to number of extrema for $x < 0$

$$g(x) = \frac{(x-1)(x-2)}{(x-3)(x-4)}$$

Number of extrema = 2 $\Rightarrow$ Total extrema = 5

2.(11) $\displaystyle \int_0^3 \{x\}^{[x]} dx = \int_0^3 \left(x-[x]\right)^{[x]} dx = \int_0^1 1.dx + \int_1^2 (x-1)\, dx + \int_2^3 (x-2)^2\, dx = \frac{11}{6}$

3.(2) $\displaystyle I = \int_3^7 \frac{\cos x^2}{\cos x^2 + \cos(10-x)^2} = \int_3^7 \frac{\cos(10-x)^2}{\cos(10-x)^2 + \cos x^2} dx$

$$2I = \int_3^7 \frac{\cos x^2 + \cos(10-x)^2}{\cos x^2 + \cos(10-x)^2} dx = \int_3^7 dx = 4$$

$\Rightarrow \quad I = 2$

4.(2) $\displaystyle \int \frac{x^{1/2}}{\sqrt{\left(a^{3/2}\right)^2 - \left(x^{3/2}\right)^2}} dx = \frac{2}{3} \sin^{-1}\left(\frac{x^{3/2}}{a^{3/2}}\right) + C$

5.(63) $\int\left(x^{20}+x^{13}+x^{6}\right)\left(2x^{21}+3x^{14}+6x^{7}\right)^{1/7}dx$

$2x^{21}+3x^{14}+6x^{7}=t$

$42\left(x^{20}+x^{13}+x^{6}\right)dx=dt$

$$\frac{1}{42}\int_0^{11}t^{1/7}dt=\left(\frac{t^{8/7}}{8/7}\times\frac{1}{42}\right)_0^{11}=\frac{1}{48}\left(t^{8/7}\right)_0^{11}=\frac{1}{48}(11)^{8/7}$$

$l=48, m=8, n=7$

$l+m+n=63$

6.(62) $A=\int_{-1}^{0}\left(x^2-3x\right)dx+\int_0^2\left(3x-x^2\right)dx$

$$\Rightarrow A=\frac{x^3}{3}-\frac{3x^2}{2}\Bigg|_{-1}^{0}+\frac{3x^2}{2}-\frac{x^3}{3}\Bigg|_0^2\Rightarrow A=\frac{11}{6}+\frac{10}{3}=\frac{31}{6}\ \therefore 12A=62$$

7.(1200) $\because$ Degree of $P(x)$ is 5 with leading coefficient one.

$\therefore$ Degree of $P'(x)$ is 4 with leading coefficient five.

$\therefore\ P'(x)=5(x-1)(x-3)(x-2)^2$

$\therefore\ P'(6)=5\times5\times3\times4^2=1200$

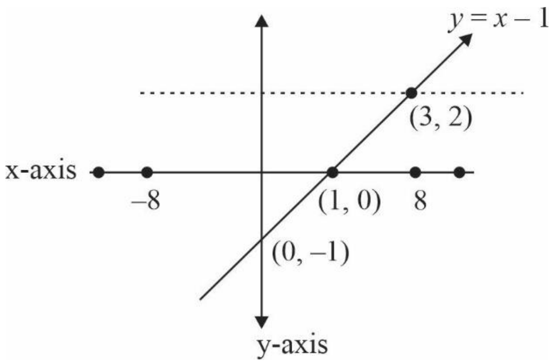
8.(5) $[x]^2=[y]^2$, for $1\le x\le 4$, is: $\{x-1\}=x-1-[x-1]=x-[x]$ $\quad\therefore$ Req $=\int_{-5}^{5}(x-[x])dx$

9.(4) For $|x|<8$, we have $0\le x^2<64$

$$\Rightarrow 0\le\frac{x^2}{64}<1\Rightarrow 2\le\frac{x^2}{64}+2<3$$

$$\therefore\left[\frac{x^2}{64}+2\right]=2,\ \text{for } |x|<8$$

Req $=\int_0^2(y+1)dy$



10.(6) $I=\int_{-2}^{2}\frac{|x^3+x|}{e^{x|x|}+1}dx=\int_{-2}^{2}\frac{|x^3+x|}{e^{-x|x|}+1}dx=\int_{-2}^{2}\frac{e^{x|x|}.|x^3+x|\,dx}{e^{x|x|}+1}$

$2I=\int_{-2}^{2}\left|x^3+x\right|dx=2\int_0^2(x^3+x)dx$

$\Rightarrow I=\int_0^2(x^3+x)dx=6$